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HOMOGENIZED PENETRATION CALCULATIONS

STEVEN B. SEGLETES
U.S. Army Research Laboratory, AMSRL-WT-TD, Aberdeen Proving Ground, Maryland 21005-5066, U.S.A.

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Abstract—A model was developed which describes the penetration of a target composed of multiple discrete elements in terms of the penetration of an equivalent single-element target of identical thickness. To achieve this aim, the effective density and target resistance terms for the equivalent target have been homogenized from the densities and target resistances of the individual discrete elements composing the original target. Though the current model may be employed to treat a target's internal air gaps in the homogenization process, the current model does nothing special to address target considerations, such as obliquity, confinement, length-to-diameter (L/D) effects, etc. Rather, the model was intentionally restricted to flat-plate-type target elements being impacted at normal incidence, to focus upon the homogenization technique itself. To avoid the need to transform the shape or velocities of the bodies in question, only techniques which strictly preserved length and time dimensions were considered. Several homogenization schemes were examined and compared to the corresponding multi-element penetration calculation. It was determined that a straightforward volume averaging of target properties is usually not sufficient to effectively simulate a multi-element target. Other techniques presented here seem to do a better job at predicting residual penetrator length and velocity, respectively. Copyright © 1996 Elsevier Science Ltd

1. INTRODUCTION

It is of general interest to develop an analytical model to describe the behavior of two nonhomogeneous bodies during impact at high obliquity and at impact velocities of several kilometers per second. One approach involves sectionally homogenizing the bodies and employing a Tate-like penetration analysis (Tate, 1967) on the resulting problem. As a first step in solving this general problem, the straightforward case of a uniform rod penetrating a multi-element target at normal incidence is considered. Effort was directed at constructing the appropriate homogenization relations for such a target configuration, in the context of a Tate penetration analysis. In such analyses, only penetrator and target densities, penetrator strength and target resistance are employed as material parameters. To avoid the need to transform the shape or velocities of the bodies in question, only techniques which strictly preserve length and time dimensions are considered.

In addition to a straightforward volume averaging technique for both density and target resistance, a density homogenization is examined which preserves penetrator erosion in the hydrodynamic limit and two different target resistance homogenizations are considered. In one, the homogenization attempts to preserve the decelerative impulse delivered to the penetrator and should thus provide a good predictor for residual penetrator velocity. In the other, the homogenization attempts to preserve eroded penetrator length, even for cases below the hydrodynamic limit. This latter technique should thus provide a reasonable predictor for residual rod length. One very interesting result that arises from these two target resistance homogenizations is that they depend upon the penetrator density, in both cases, and upon the penetrator strength, in the latter case. This result is similar to the conclusion of Wright and Frank (1988) that "R is not simply a measure of target hardness, but it involves characteristics of the rod and of the specific collision under consideration." However, their conclusion pertains to the balance laws within a homogeneous target, whereas the current conclusion arises from the nature of multiplate-target homogenization. Though such a dependence of target resistance on penetrator parameters may seem intellectually unsatisfying, it may actually be necessary in order to achieve a proper homogenization of material properties.

2. THEORY

In the following discussion, the normal impact of a uniform penetrator of specified density, ρ_R, length, L, strength, Y, and velocity, V_0, upon a multi-element target of thickness, t, shall be considered. The target is considered to be composed of n layers, each of density ρ_i, thickness t_i, and target resistance, R_i. An air gap may comprise one or more of these target layers. The goal of this exercise is to formulate an equivalent target of identical total thickness, t, whose effect upon the eroding penetrator closely resembles that of the original multi-element target. Since a fundamental understanding of target homogenization is desired, comparisons of the multi-element-target solution to various homogenized solutions will be accomplished with the original Tate penetration model (Tate, 1967). Higher order effects of obliquity, confinement, L/D, etc. will not be considered here, since they could otherwise obscure conclusions and trends of the homogenization modeling itself. Additionally, it is not the intent of this paper to debate the merits of utilizing a Tate solution for various applications. Rather, the intent is to obtain the best homogenization technique, given that the Tate approach will be employed.

Below the ballistic limit, penetrator response can be a strong function of the target element stacking order. Homogenization algorithms will, by their nature, fail to capture these variations resulting from the stacking arrangement. Thus, to avoid confusing variations due to the target-element stacking arrangement with intrinsic differences in the homogenization algorithms themselves, discussion will be limited to only those cases above the ballistic limit (where target perforation occurs). Even above the ballistic limit, penetrator response will be mildly affected by target element stacking order. However, this influence rapidly diminishes with increasing striking velocity and quickly becomes negligible.

2.1. *Volume-averaged parameters*

Volume averaging (a.k.a. the rule of mixtures) may seem a good, logical, first step at approaching the homogenization problem. Continuity of target mass is automatically satisfied if density is volume averaged, which seems appealing. For a one-dimensional approach, volume averaging is equivalent to length averaging. Thus, the homogenization of a parameter, for example ρ, associated with each target element i, would be volume averaged by weighting each element's ρ_i by the element thickness, to give

$$\bar{\rho} = \frac{\sum_{i=1}^{n} \rho_i t_i}{\sum_{i=1}^{n} t_i}. \tag{1}$$

The barred quantity will be uniformly used in this paper to denote a volume-averaged homogenization. For the current discussion, the homogenization of the type given in (1) may also be identically performed on target resistance, merely by substituting R for ρ. As will be shown in subsequent analyses, the volume-averaged homogenization technique is not a particularly accurate method to describe residual penetrator lengths and velocities, especially at higher impact velocities.

2.2. *Hydrodynamic erosion homogenization for density, ρ_H*

An homogenization method for density is offered here which has the virtue of predicting the proper residual penetrator length in the hydrodynamic limit. As striking velocity increases, the influence of both penetrator strength and target resistance is monotonically lessened, and erosion is governed solely by density considerations.

Consider the case of hydrodynamic penetration in order to develop this formulation. In the hydrodynamic limit, a Bernoulli balance indicates that an increment of penetrator erosion equals μ times the increment of target penetration, the constant, μ, being given by the square root of the target to penetrator density ratio, $\mu = \sqrt{\rho/\rho_R}$. If we use this hydrodynamic limiting case to homogenize the penetrator/target erosion process, we may equate the total length of eroded penetrator through the homogenized target to the incremental sum of eroded lengths through each of the individual target elements (i.e., $\Delta L = \Sigma \Delta L_i$). Substituting terms gives:

$$\mu_H t = \sum_{i=1}^{n} \mu_i t_i, \tag{2}$$

where the 'H' subscript represents this "hydrodynamic" homogenization which preserves length erosion in the hydrodynamic limit. Expressing homogenized target thickness, t, as the sum of the original target element thicknesses, the solution for μ_H becomes

$$\mu_H = \frac{\sum_{i=1}^{n} \mu_i t_i}{\sum_{i=1}^{n} t_i}. \tag{3}$$

Reducing μ back to its density primitives permits (3) to be expressed in terms of the hydrodynamic homogenized density:

$$\sqrt{\rho_H} = \frac{\sum_{i=1}^{n} \sqrt{\rho_i}\, t_i}{\sum_{i=1}^{n} t_i}. \tag{4}$$

This definition of density is the relevant metric for hydrodynamic erosion, though it clearly differs from the volume-averaged density of (1), and not by a trivial amount. For example, considering a baseline target composed of equal thicknesses of steel ($\rho = 7.8$ g/cm^3) and air ($\rho \simeq 0$), the two homogenized densities are given as $\bar{\rho} = 3.9$ and $\rho_H = 1.95$. In the hydrodynamic limit (assuming enough penetrator length to permit perforation), the volume-averaged target density would predict penetrator erosion 41% larger than the actual length consumption.

For lower impact velocities, penetration behavior is no longer hydrodynamic, and strength considerations will play an increasingly important role. Though it would be possible to construct a model in which both homogenized density and target resistance were both functions of impact velocity, such complications can hopefully be avoided. As such, target resistance homogenizations to be considered below will be cast in a context where the homogenized density is a function of geometry only and thus not a function of impact velocity.

2.3. *'Impulse' homogenization for target resistance, R_V*

To develop one estimate of what an homogenized resistance might be, consider the function of the resistance terms, R and Y, in the modified Bernoulli equation:

$$\tfrac{1}{2}\rho_R(V-U)^2 + Y = \tfrac{1}{2}\rho U^2 + R. \tag{5}$$

These terms modify the stress (and thus, force) in the momentum-based Bernoulli equation, thereby modifying the impulse imparted to the projectile and target. If one speculates that the impulse, $\int F\,dt$, delivered to the penetrator by the target resistance should remain unaffected by the homogenization process, then it will be necessary to obtain the time spent penetrating each element of the target laminate. A first order approximation is likewise available from hydrodynamic considerations. However, the penetration time estimated in this manner will be in slight error when the penetration conditions are short of hydrodynamic. Proceeding nonetheless, the hydrodynamic penetration rate, given as

$$U = \frac{V}{1+\mu}, \tag{6}$$

may be employed to approximate the time, τ_i, spent penetrating each element. This time is given as

$$\tau_i = t_i/U_i \simeq t_i(1+\mu_i)/V_0. \tag{7}$$

Using this per element penetration time in the impulse equation, the impulse-homogenized target resistance, R_V, is given by

$$R_V = \frac{\sum_{i=1}^{n} R_i(1+\mu_i)t_i}{\sum_{i=1}^{n} (1+\mu_i)t_i}. \tag{8}$$

Since μ_i is a function of the penetrator density, the relationship (8) has the complication of requiring the penetrator density to compute the target resistance R_V. Because this homogenization strove to preserve impulse delivered by the target, one might hope that such a formulation would provide a reasonably good predictor for residual penetrator velocity existing a target.

2.4. *'Erosion' homogenization for target resistance*, R_L

One may derive a target resistance homogenization that strives to conserve eroded penetrator length, rather than impulse delivered to the penetrator. Consider the penetrator-erosion and the modified-Bernoulli equations, as the penetrator is traversing target element i:

$$\dot{L}_i = U_i - V_i, \tag{9}$$

$$\tfrac{1}{2}\rho_R(V_i - U_i)^2 + Y = \tfrac{1}{2}\rho_i U_i^2 + R_i. \tag{10}$$

Solve for $\dot{L}_i$ to eliminate V_i:

$$\dot{L}_i = -\sqrt{\frac{2(R_i - Y)}{\rho_R} + \frac{\rho_i}{\rho_R}U_i^2}. \tag{11}$$

If target element i is of thickness t_i, and the time required to penetrate this element is τ_i, then the amount of penetrator erosion occurring in element i is

$$\Delta L_i = \dot{L}_i \tau_i, \tag{12}$$

where the time spent penetrating element i may be approximated, as before, through the use of (7). In this homogenization approach, the sum of the eroded penetrator lengths through each target element, should equal the total eroded length through the homogenized target, or $\Delta L = \Sigma \Delta L_i$. Substitution gives

$$\frac{\dot{L}_{eff}}{U_{eff}} \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} \frac{\dot{L}_i}{U_i} t_i, \tag{13}$$

where the sum of the individual target element thicknesses has been substituted for the total homogenized target thickness. Note that, from (11) above,

$$\dot{L}_{eff} = -\sqrt{\frac{2(R_L - Y)}{\rho_R} + \frac{\rho_{eff}}{\rho_R} U_{eff}^2}. \tag{14}$$

The term "ρ_{eff}" represents the homogenized density being used in conjunction with this target resistance homogenization. We will choose the hydrodynamic density homogenization, ρ_H, to be that effective density for the remainder of this discussion, or $\rho_{eff} = \rho_H$. It immediately follows that the effective penetration rate, U_{eff}, is also based on the hydrodynamic density homogenization (and will thus be called U_H). It is given as

$$U_H \simeq \frac{V_o}{1+\mu_H} = \frac{V_0}{1+\sqrt{\rho_H/\rho_R}}. \tag{15}$$

We seek an homogenized approximation to the term "R_L". Substitute (11), (14), and (15) into (13) and solve for R_L:

$$R_L = Y + \left(\frac{U_H}{\sum_{i=1}^{n} t_i} \sum_{i=1}^{n} t_i \sqrt{\frac{(R_i - Y)}{U_i^2} + \frac{\rho_i}{2}} \right)^2 - \frac{\rho_H U_H^2}{2}. \tag{16}$$

This homogenized resistance is nominally a function of not only target geometry, but also of penetrator density, strength, and impact velocity. Though R_L may, at first, seem to be a strong function of U_H, and thus striking velocity, V_0, this conclusion turns out not to be the case, and in fact R_L quickly asymptotes to a constant value with increasing striking velocity, due to a cancelling of terms.

To obtain this asymptote, use (4) to replace the homogenized hydrodynamic density with its raw constituents, and express (16) as follows:

$$R_L = Y + \left[\left(\sum_{i=1}^{n} \sqrt{a_i + Kb_i} \right)^2 - \left(\sum_{i=1}^{n} \sqrt{Kb_i} \right)^2 \right] \Big/ \left(\sum_{i=1}^{n} t_i \right)^2, \tag{17}$$

where

$$a_i = \frac{(R_i - Y) U_H^2 t_i^2}{U_i^2}$$

$$b_i = \frac{\rho_i t_i^2}{2}, \quad \text{and}$$

$$K = U_H^2.$$

Both U_H and U_i are directly proportional to striking velocity V_0. Their ratio is therefore independent of striking velocity. Thus, terms a_i and b_i are composed purely of geometrical and material property considerations independent of striking velocity. The goal then becomes to show that (17) is independent of K as K, the only velocity dependent term, becomes sufficiently large.

Consider, thus, the evaluation of the limit

$$\lim_{k \to \infty} \left[\left(\sum_{i=1}^{n} \sqrt{a_i + Kb_i} \right)^2 - \left(\sum_{i=1}^{n} \sqrt{Kb_i} \right)^2 \right]. \tag{18}$$

Factor out a K to obtain

$$\lim_{K\to\infty} K\left[\left(\sum_{i=1}^{n}\sqrt{\frac{a_i}{K}+b_i}\right)^2-\left(\sum_{i=1}^{n}\sqrt{b_i}\right)^2\right]. \tag{19}$$

For large K (i.e., large impact velocity), the first term may be approximated in a first order binomial expansion as

$$\sqrt{\frac{a_i}{K}+b_i} \simeq \sqrt{b_i}+\frac{a_i}{2K\sqrt{b_i}}+\cdots. \tag{20}$$

Using this two-term expansion, the limit becomes

$$\lim_{K\to\infty} K\left[\left(\sum_{i=1}^{n}\sqrt{b_i}+\frac{a_i}{2K\sqrt{b_i}}\right)^2-\left(\sum_{i=1}^{n}\sqrt{b_i}\right)^2\right]. \tag{21}$$

This limit is a difference of squares and can thus be expressed as

$$\lim_{K\to\infty} K\left[\left(\sum_{i=1}^{n}\sqrt{b_i}+\frac{a_i}{2K\sqrt{b_i}}+\sqrt{b_i}\right)\left(\sum_{i=1}^{n}\sqrt{b_i}+\frac{a_i}{2K\sqrt{b_i}}-\sqrt{b_i}\right)\right]. \tag{22}$$

For large K, the first term becomes

$$\left(2\sum_{i=1}^{n}\sqrt{b_i}\right)$$

and the second becomes

$$\left(\sum_{i=1}^{n}\frac{a_i}{2K\sqrt{b_i}}\right).$$

The limit thus becomes independent of K, and is given by

$$\lim_{K\to\infty}\left[\left(\sum_{i=1}^{n}\sqrt{a_i+Kb_i}\right)^2-\left(\sum_{i=1}^{n}\sqrt{Kb_i}\right)^2\right]=\left(\sum_{i=1}^{n}\sqrt{b_i}\right)\left(\sum_{i=1}^{n}\frac{a_i}{\sqrt{b_i}}\right). \tag{23}$$

With the use of this limit and appropriate substitution, velocities may be removed from eqn (17) to obtain finally the asymptotic value of R_L, which we will call $R_{L\infty}$:

$$R_{L\infty} = Y+\frac{\sqrt{\rho_H}}{(1+\sqrt{\rho_H/\rho_R})^2}\,\frac{\displaystyle\sum_{i=1}^{n}\frac{(R_i-Y)(1+\sqrt{\rho_i/\rho_R})^2 t_i}{\sqrt{\rho_i}}}{\displaystyle\sum_{i=1}^{n} t_i}. \tag{24}$$

To justify the use of this homogenization asymptote (24), examine how the results of R_L, given by (16), and $R_{L\infty}$ compare for a representative impact condition involving the

Table 1. Representative penetrator/target geometry used to compare actual and asymptotic length-based, homogenized target resistances, R_L and R_{Lx}, respectively

ρ_R (kg/m³)	Y (GPa)	L_0 (m)
8900	0.5	1.0
ρ_i (kg/m³)	R_i (GPa)	t_i (m)
7800	1.0	0.3
0	0	0.5
2800	2.0	0.3
4800	3.0	0.3

multi-element target described in Table 1. The comparison of R_L and R_{Lx} for this case is depicted in Fig. 1. At all velocities, the functions are within 10% of each other, and at velocities of ballistic interest (for example, greater than 1600 m/s), the functions are within 3% of each other. Though the merits of the R_L homogenization will be explored subsequently, the use of the asymptotic expression to approximate the formulation seems justifiable. Though using the asymptote expressed in (24) makes homogenized target resistance independent of penetrator striking velocity, the expression is quite dependent upon penetrator density and strength. For all cases of (24) studied to date, strengthening a penetrator (increasing Y) has the net effect of creating an apparently stronger homogenized target (increasing R_{Lx}).

3. RESULTS

In actual applications of the penetration equations, the target resistance usually exceeds the penetrator strength. An explanation of this tendency is offered by Wright and Frank [1988], who show that the target resistance, R, is justifiably composed of a variety of terms, the net effect of which is to make the value of R several times the actual target yield strength. Thus, the word "resistance" and not "strength" is used to describe this term.

Nonetheless, to better assess the value of the various homogenization techniques discussed herein, cases will be examined in which the rod strength exceeds the target resistance, as well as the (more usual) converse. Additionally, for both of these situations, a case will be studied for a high-density penetrator relative to the target elements, as well as the case of a low-density penetrator relative to the target elements.

These four permutations are given as follows: case I: high-density, weak penetrator; case II: high-density, strong penetrator; case III: low-density, weak penetrator; and case IV: low-density, strong penetrator. The penetrator/target geometries illustratively chosen

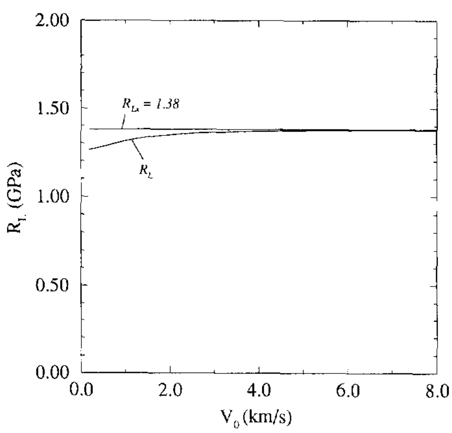

Fig. 1. A comparison of the eroded-length homogenization for target resistance, eqn (16), with its asymptote, eqn (24), for the representative impact conditions described in Table 1.

for these four cases are listed in Tables 2–5, along with the various homogenized representations. For the low-density penetrators, the target dimensions have been scaled back to permit penetrator perforation. Because the volume fraction of each target element is the same in all four cases, volume-averaged properties (density and target resistance) in all cases remain unchanged. The "hydrodynamic" homogenized density, though numerically different from the volume-averaged density, is also constant for all cases. Since homogenized strength R_V is a function of penetrator density, its value for cases I and II will take on one value, and for cases III and IV, a different value. The homogenized strength R_{Lx}, on the

Table 2. Penetrator/target geometry (case I: high-density, weak penetrator) used to compare homogenization techniques

ρ_R (kg/m^3)	Y (GPa)	L_0 (m)
8900	0.5	1.0
ρ_i (kg.m^3)	R_i (GPa)	t_i (m)
7800	1.0	0.3
0	0	0.5
2800	2.0	0.3
4800	3.0	0.3
$\bar{\rho} = 3300$	$\bar{R} = 1.29$	$t = 1.4$
$\rho_H = 2035$	$R_V = 1.49$	$t = 1.4$
$\rho_H = 2035$	$R_{Lx} = 1.38$	$t = 1.4$

Table 3. penetrator/target geometry (case II: high-density, strong penetrator) used to compare homogenization techniques

ρ_R (kg/m^3)	Y (GPa)	L_0 (m)
8900	5.0	1.0
ρ_i (kg/m^3)	R_i (GPa)	t_i (m)
7800	1.0	0.3
0	0	0.5
2800	2.0	0.3
4800	3.0	0.3
$\bar{\rho} = 3300$	$\bar{R} = 1.29$	$t = 1.4$
$\rho_H = 2035$	$R_V = 1.49$	$t = 1.4$
$\rho_H = 2035$	$R_{Lx} = 3.25$	$t = 1.4$

Table 4. Penetrator/target geometry (case III: low-density, weak penetrator) used to compare homogenization techniques. Target element thicknesses scaled down to permit perforation

ρ_R (kg/m^3)	Y (GPa)	L_0 (m)
2700	0.5	1.0
ρ_i (kg/m^3)	R_i (GPa)	t_i (m)
7800	1.0	0.2
0	0	0.333
2800	2.0	0.2
4800	3.0	0.2
$\bar{\rho} = 3300$	$\bar{R} = 1.29$	$t = 0.933$
$\rho_H = 2035$	$R_V = 1.58$	$t = 0.933$
$\rho_H = 2035$	$R_{Lx} = 1.48$	$t = 0.933$

Table 5. Penetrator/target geometry (case IV: low-density, strong penetrator) used to compare homogenization techniques. Target element thicknesses scaled down to permit perforation

ρ_R (kg/m^3)	Y (GPa)	L_0 (m)
2700	5.0	1.0
ρ_i (kg/m^3)	R_i (GPa)	t_i (m)
7800	1.0	0.2
0	0	0.333
2800	2.0	0.2
4800	3.0	0.2
$\bar{\rho} = 3300$	$\bar{R} = 1.29$	$t = 0.933$
$\rho_H = 2035$	$R_V = 1.58$	$t = 0.933$
$\rho_H = 2035$	$R_{Lx} = 3.01$	$t = 0.933$

other hand, is a function of both penetrator density and strength. Thus, its value changes for each of the four cases. Furthermore, its value increases significantly over the other resistance homogenization formulations when the penetrator strength is large compared to target element resistances (e.g., cases II and IV).

Figures 2–5 depict the residual penetrator length exiting the targets as a function of impact velocity for each of the cases described previously. In each figure, the benchmark curve is shown for the actual multi-element penetration solution, as well as curves for the volume-averaged, impulse, and erosion homogenizations. In all of these figures, residual length data is shown only at velocities above the limit (perforation) velocity for each of the targets. Figures 6–9 show similar comparisons for the same four impact geometry cases, except that residual velocity, not length, is depicted. Solutions presented in the figures were achieved using the technique described by Walters and Segletes [1991].

Though the four cases examined here by no means compose an exhaustive cross section of possible penetrator/target conditions, they do explore some key variations in the parameter space of the relevant variables. In a paper of this nature, brevity requires a limit on the number of cases presented. Though these cases represent the results obtained to date, it is clearly possible that some of the conclusions drawn may have to be modified as the solution parameter space is further explored. The following observations may be drawn upon examination of the four cases studied:

(1) Volume averaging as an homogenization technique is likely to be a poor predictor of penetration, especially when results are taken over a wide impact velocity range (up to and including hypervelocity).

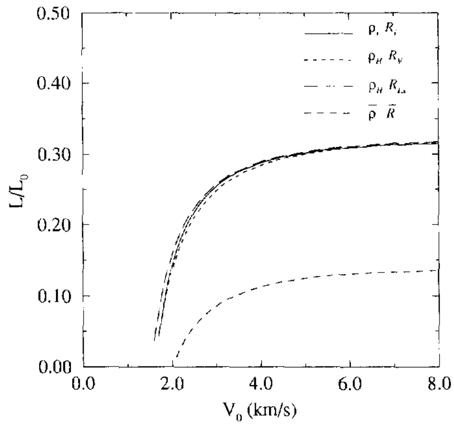

Fig. 2. Residual penetrator length vs impact velocity for case I impact conditions of high density, weak penetrator (Table 2). Plot depicts multiplate-target solution plus three homogenization alternatives.

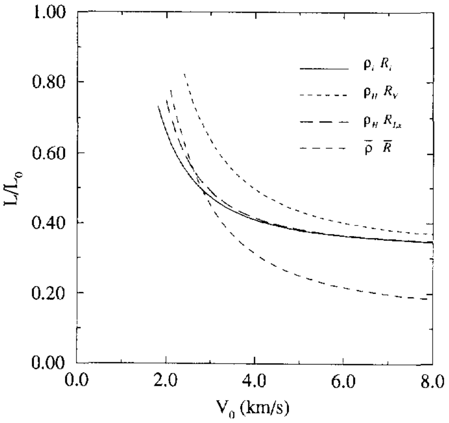

Fig. 3. Residual penetrator length vs impact velocity for case II impact conditions of high-density, strong penetrator (Table 3). Plot depicts multiplate-target solution plus three homogenization alternatives.

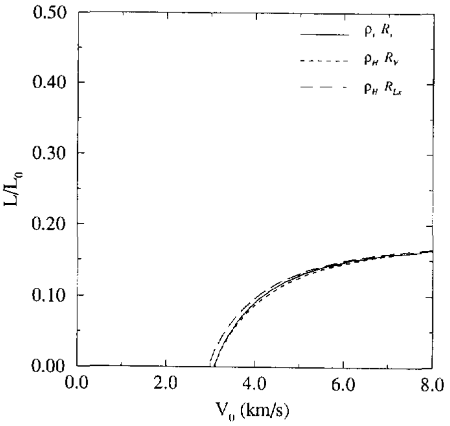

Fig. 4. Residual penetrator length vs impact velocity for low-density, weak penetrator (Table 4). Plot depicts multiplate-target solution plus two homogenization alternatives (volume-averaged target fails to perforate).

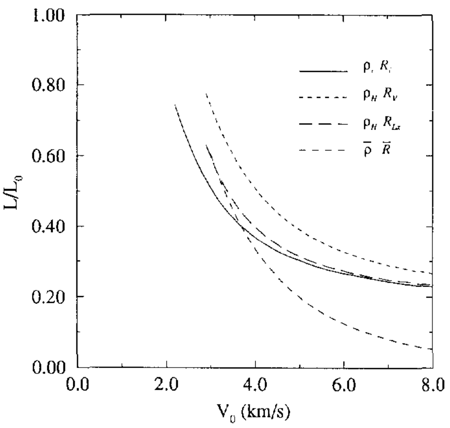

Fig. 5. Residual penetrator length vs impact velocity for case IV impact conditions of low-density, strong penetrator (Table 5). Plot depicts multiplate-target solution plus three homogenization alternatives.

(2) As impact velocity increases, the density homogenization becomes the primary determinant of penetrator erosion.

(3) For homogenization schemes which preserve measures of length, the hydrodynamic density homogenization, ρ_H, is the only homogenization which assures a correct accounting of penetrator erosion in the hydrodynamic limit.

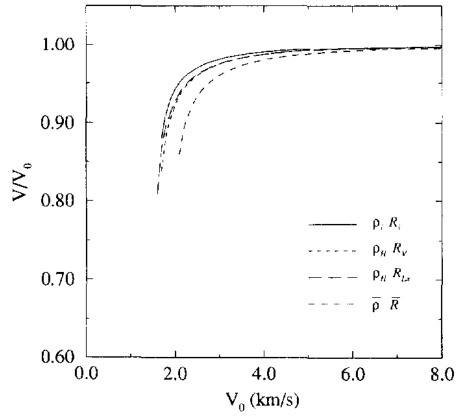

Fig. 6. Residual penetrator velocity vs impact velocity for case I impact conditions of high-density, weak penetrator (Table 2). Plot depicts multiplate-target solution plus three homogenization alternatives.

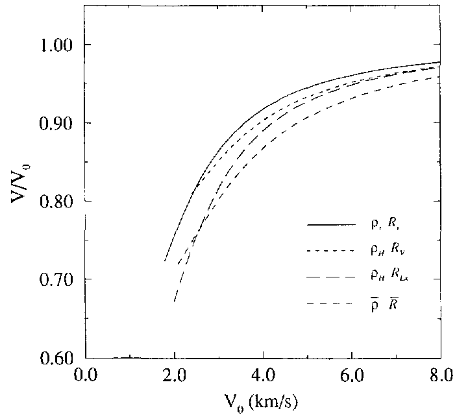

Fig. 7. Residual penetrator velocity vs impact velocity for case II impact conditions of high-density, strong penetrator (Table 3). Plot depicts multiplate-target solution plus three homogenization alternatives.

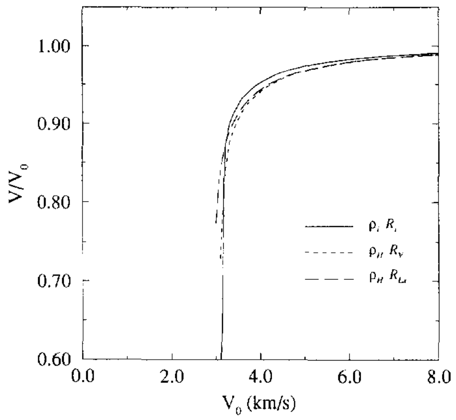

Fig. 8. Residual penetrator velocity vs impact velocity for low-density, weak penetrator (Table 4). Plot depicts multiplate-target solution plus two homogenization alternatives (volume-averaged target fails to perforate).

(4) For the material properties and target geometry covered in the cases studied, volume-averaged density always overestimated the amount of penetrator erosion in the high-velocity limit. The magnitude of the error in eroded length, at the hypervelocity limit, can be obtained by the formula $\sqrt{\bar{\rho}/\rho_H}-1$, which for the cases studied gives an error of 27%.

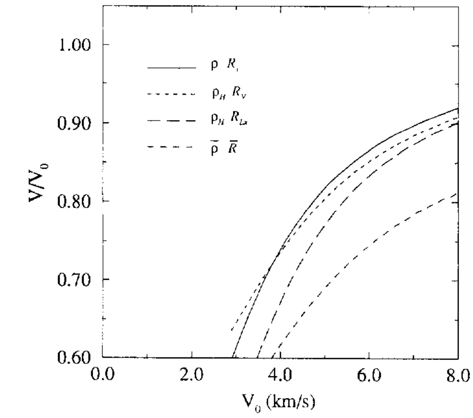

Fig. 9. Residual penetrator velocity vs impact velocity for case IV impact conditions of low-density, strong penetrator (Table 5). Plot depicts multiplate-target solution plus three homogenization alternatives.

(5) For penetrators low in strength compared to target resistances ($Y \ll R_i$, as in cases I and III), both of the alternate strength homogenizations proposed in this paper, R_V and R_{Lx}, produce very similar results which compare favorably to the actual multiplate Tate solution in both residual penetrator length and residual velocity. The impulse homogenization, R_V, might be slightly superior under these conditions. Because target resistance, R, is generally several times the value of target material yield strength, the low-strength penetrator conditions described by cases I and III are the norm.

(6) For penetrator comparable or higher in strength compared to target resistances ($Y \geqslant R_i$, as in cases II and IV), the erosion homogenization, R_{Lx}, is by far the best predictor of residual penetrator length of the schemes studied. Furthermore, it is the only homogenization scheme studied which is able to produce an homogenized target resistance larger than any of the constituent resistances in the target array. Note that R_{Lx} partially depends upon penetrator strength, Y, and that a high penetrator strength will boost the value of R_{Lx}. The need for this exaggerated homogenized target resistance, for conditions involving high penetrator strengths, leads to the important conclusion that a target composed of multiple discrete elements will respond in kind to the strengthening of the penetrator. Even though the individual target elements remain unstrengthened, the aggregate target behaves as if it were stronger!

(7) For penetrators comparable or higher in strength compared to target resistances ($Y \geqslant R_i$, as in cases II and IV), the impulse homogenization, R_V, is the best predictor of residual velocity of the schemes studied.

4. CONCLUSIONS

This paper describes and examines several homogenization approaches to a multi-element target penetration problem, in the context of the Tate penetration model. To keep this initial analysis into homogenization techniques straightforward, only cases consisting of a uniform rod penetrating into a multi-element target at normal incidence were considered. In the various schemes examined, length and time dimensions are preserved, so that physical dimensions and velocities are not affected by the homogenization procedure. Such length and time preserving schemes might be useful when it is desired to sectionally homogenize a complex body, such that the overall body dimensions remain unchanged, while at the same time, the internal detail of the body is replaced by a simpler, homogenized representation.

In the penetration analysis considered here, the material parameters of target density and target resistance are subject to homogenization. In addition to straight volume-averaging homogenization for both density and resistance, a "hydrodynamic" density homogenization is considered which explicitly preserves penetrator length erosion in the hypervelocity limit. Also, two target resistance homogenizations, so-called "impulse" and "erosion" homogenizations, are explored. These two schemes homogenize target resistance in a fashion which attempts to preserve, in the former case, impulse delivered to the decelerating penetrator, and in the latter, eroded length at velocities below the hydrodynamic limit.

It is clear from the physics of the Bernoulli equation (which the Tate equations approach in the high-velocity limit) that a volume-averaged density homogenization will yield incorrect predictions of penetration at hypervelocity impact speeds. The magnitude of the error will depend upon the specific target geometry and material properties in question, but will be over 40% for the simple case of a target composed of equal volumes of any material and void. For the more general cases studied in this report, the error was 27%. In general, any targets which have a significant percentage of air (void) will tend to accentuate the error produced by the volume-averaged density homogenization.

At lower impact velocities, the homogenization of target resistance becomes increasingly important, though its effect is still coupled with that of the density homogenization. If one accepts the notion of using a density homogenization independent of impact velocities, then in order to capture target behavior over the wide range of material property space, the target resistance homogenization must become a function of the penetrator properties. The "erosion" homogenization for target resistance described in this report demonstrates this penetrator dependence clearly, not only by the explicit occurrence of penetrator strength, Y, in eqn (24), but also by the fact that the homogenized resistance can exceed the constituent resistances of each of the target elements, if Y is correspondingly large.

For situations where the penetrator strength is small relative to target resistance, both the "impulse" and "erosion" homogenizations for strength produce similar results, matching the actual multi-element target solution well. When the penetrator strength is relatively large, the "impulse" homogenization does the best job of predicting residual penetrator velocity exiting a finite target, while "erosion" homogenization does the best job of predicting residual penetrator length under the same conditions. Since actual penetration capability (at the velocities of interest) is more strongly dependent upon penetrator length than velocity, the "erosion" resistance would be expected to provide the best penetration prediction capability of the schemes studied.

There are many opportunities for future work in this area. The most obvious area would be to consider homogenization schemes for heterogeneous penetrators, as opposed to targets. Since the target homogenization schemes examined here, in some cases, depend upon penetrator properties, the problem of simultaneously homogenizing heterogeneous penetrator and target poses additional challenges. Generalizing the homogenization schemes to handle arbitrary 3-D geometries, rather than a 1-D approximation (rod impacting multiple flat-plate target configuration) is also a worthwhile endeavor. As a first step in this direction, one might replace the target-element thicknesses in the current methodology equations with the package thickness multipled by the target-element volume fraction. Finally, and perhaps of greater importance than the current effort, is the issue of how to homogenize lateral crater size and/or lateral damage extent.

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